William Penn Progression in Calculation

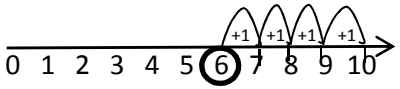
+ Stages in Addition+				
Early recording	Jottings and informal methods			Formal written methods
Using representations	Number lines	Partitioning	Number lines	Vertical layout

Using representations

6 + 4 = 10

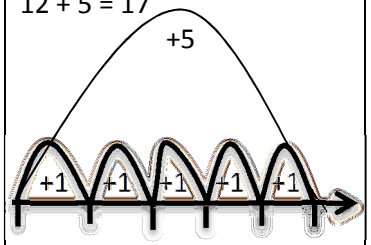
Counting on using pictures

Counting on using a number line:

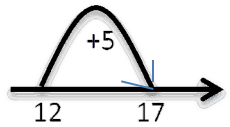


Number lines

12 + 5 = 17



Empty number line:



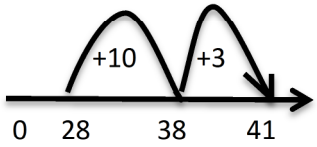
Partitioning

13 + 28 = 41

Adding tens first then units
And recombining:

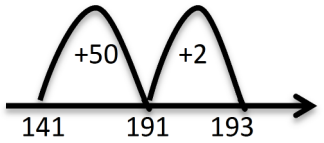
20 + 10 = 30
8 + 3 = 11
30 + 11 = 41

Using a number line with partitioning, add on tens then units:



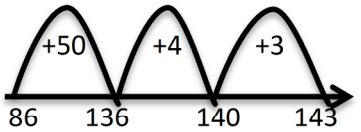
Number lines

141 + 52 = 193



Bridging through 10:

86 + 57 = 143



Vertical layout

1. Introduce with expanded working, by partitioning the numbers:
   Start with 2 digit numbers:

47 + 76

40 + 7
70 + 6
110 + 13 = 123

Extend to 3 digit numbers:
368 + 493

300 + 60 + 8
400 + 90 + 3
700 + 150 + 11 = 861

2. Now use expanded working but without partitioning:

H T U	H T U
4 7	3 6 8
+ 7 6	+ 4 9 3
1 3 (7+ 6)	1 1(8+3)
1 1 0(40+70)	1 5 0(60+90)
1 2 3	7 0 0(300+400)
	8 6 1

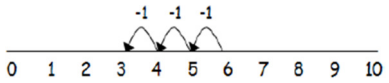
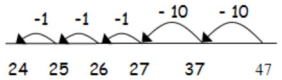
-Brackets could be removed in time.

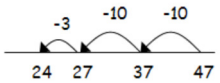
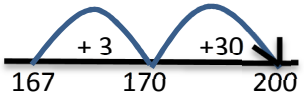
3. Final stage – contract the working to a compact, efficient form

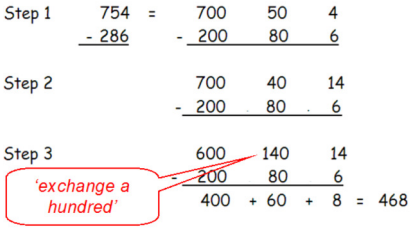
H T U	H T U
4 7	3 6 8
+ 7 6	+ 4 9 3
1 2 3	8 6 1
1 1	1 1

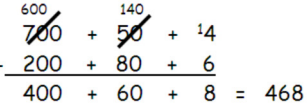
4. Extend to larger numbers and decimals.

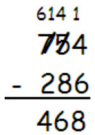
William Penn Progression in Calculation

<table>
<tr><th colspan="4">-Stages in Subtraction-</th></tr>
<tr><th>Early recording</th><th colspan="2">Jottings and informal methods</th><th>Formal methods</th></tr>
<tr><th>Using representations</th><th>Counting back with number lines</th><th>Counting up with number lines</th><th>Vertical layout</th></tr>
<tr>
<td>Children count and point using objects, physically moving them. They respond to questions like – ‘I have 3 balloons, 2 burst. How many are left?’
 
Count back in ones.
6 – 3 = 3
 
</td>
<td>Counting back in 10s and ones.
47 - 23 = 24
 

Develop to subtracting 10s and units.
47 - 23 = 24
 
</td>
<td>Using larger numbers, use finding the difference to count up:
200 – 167 = 33
 

Use for money:
£10 - £6.84 = £3.16
 
</td>
<td>- Expanded column subtraction:
89 = 80 9
- 57 - 50 7
30 + 2 = 32
- Move on to examples where exchange occurs. Use Dienes and 100, 10 and 1 labelled counters to support understanding:
 

- Move towards:
 

- Move to compact written methods and larger numbers:
 

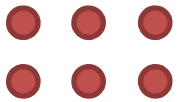
 
- Use compact written methods for decimals:
  </td>
</tr>
</table>

William Penn Progression in Calculation

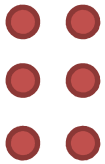
x Stages in Multiplication x

Early recording – Using representations

3 lots of 2 is 6



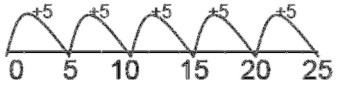
2 lots of 3 is 6



Informal methods – Repeated Addition

Counting in twos, fives, etc

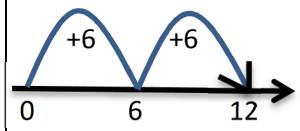
5 x 5 = 5 + 5 + 5 + 5 + 5



Supported by apparatus.

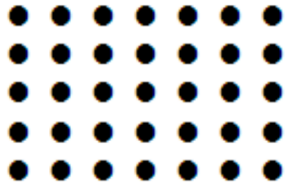
Or

2 x 6
6 + 6 = 12



Informal methods – Arrays

7 x 5 = 35



5 x 7 = 35

Children also understand the commutative law:

Use examples such as

6 x 4 = 4 x 6

Towards formal methods – Grid Method

1. Start by multiplying 2 digit numbers by 1 digit numbers:

38 x 7

x	30	8
7	210	56

= 266

(Children use knowledge that 7 x 3 = 21 and then multiply 21 by 10 to work out answer to 7 x 30).

Move on to multiplying 3 digit numbers by 1 digit numbers.

425 x 6

x	400	20	5
6	2400	120	30

= 2550

(Children use knowledge that 6 x 4 = 24 and then multiply 24 by 100 to work out answer to 6 x 400).

2. Extend to multiplying by 2 digit numbers and decimals:

56 x 27

x	50	6
20	1000	120
7	350	42

= 1120
+ 392
1512
1

Formal methods – Column Multiplication

1. Start with expanded notation working ensuring clear links to the grid method:

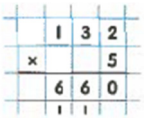
63 x 34

63	x
34	
12	(4x3)
240	(4x60)
90	(30x3)
+ 1800	(30x60)
2142	

x	60	3
30	1800	90
4	240	12

1890
+ 252
2142

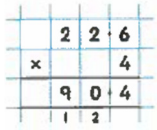
2. Once children are confident, move to short multiplication. Continue to make links with the grid method:



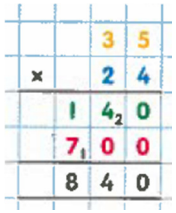
3. Move to short multiplication with decimal numbers. Continue to make links with grid strategies):

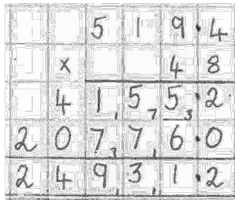
x	20	2	0·6
4	80	8	2·4

80 + 8 + 2·4 = 90·4



4. Extend to long multiplication with larger numbers and decimals:





William Penn Progression in Calculation

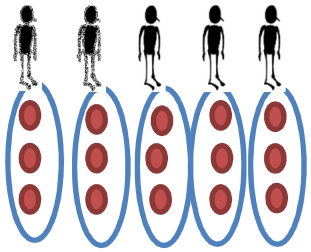
÷ Stages in Division ÷

Informal methods	Informal methods	Towards formal methods	Formal methods
Sharing	**Grouping**	**Chunking**	**Bus Stop**

Sharing

15 ÷ 5 = 3

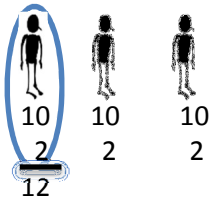
Question: 5 people share 15 sweets equally. How many does each person get?



Each child gets 3 sweets.

36 ÷ 3 = 12

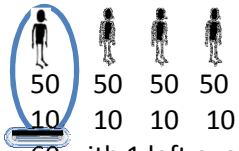
Question: A class of 36 split into 3 equal teams. How many are in each team?



10	10	10	3 x 10 = 30
2	2	2	3 x 2 = 6
12			36

Each team has 12 children in it.

241 ÷ 4 = 60 r 1

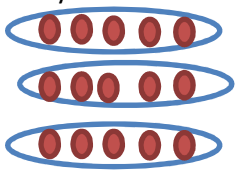


50	50	50	50	4 x 50 = 200
10	10	10	10	4 x 10 = 40
60 with 1 left over				240

Grouping

15 ÷ 5 = 3

Question: How many bags of 5 sweets can you make with 15 sweets?



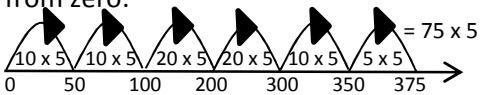
Grouping on a number line:

24 ÷ 4 =6

Question: How many groups of 4 children in a class of 24?

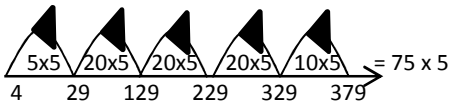
379 ÷ 5 = 75 r 4

Grouping towards the target number from zero:



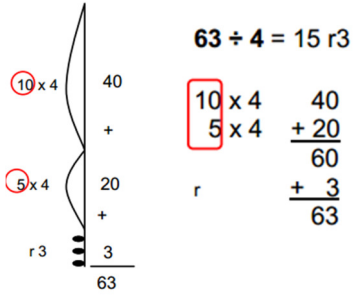
or

Grouping backwards from the target number towards zero:



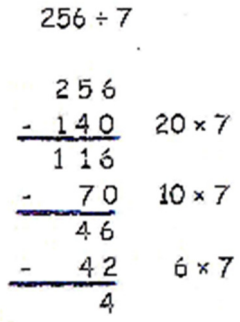
Chunking

The written methods used are based on the idea of repeated subtraction eg to solve 20 ÷ 5 we can think of it as 20 – 5 – 5 – 5 – 5. Five multiplied by 4 is 20 or there are four fives in twenty. For bigger numbers we need to take off bigger chunks in order for this to be an efficient method.



Develop this strategy with the vertical number line.

Children can then move on to whatever size chunks they feel confident with.



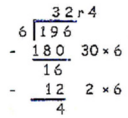
= 36 r 4

Bus Stop

This final stage contracts the working to a compact, efficient form:

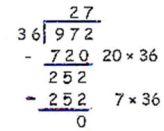
196 ÷ 4 = 32 r 4

This can be represented using place value counters and also extended to use fractions in the remainders:

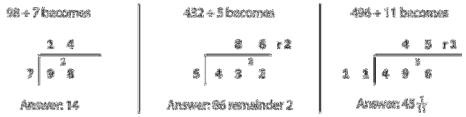


Children aim to choose the largest possible multiples of 100, 10 and then as 1 as appropriate:

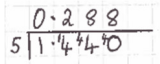
972 ÷ 36 = 27



Move to short division with the remainder as a whole number or fraction.



Next, move to short division with decimals in contexts:



Extend to long division with the remainders as a whole number, fraction or a decimal: